DISPERSIVE E.M. CORRECTIONS TO πN SCATTERING LENGTHS

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See hep-ph/0503277 and hep-ph/0504258

Part of a broader study of e.m. threshold corrections with B. Loiseau and S. Wycech PLB B594 (2004) 76

Outline

1. Background and motivation
2. Framework
3. Prototype dispersive term

$\pi^- p \rightarrow \gamma n \rightarrow \pi^- p$

4. Δ contributions
5. Results and implications
6. Comparison to present ChPT
7. Summary and outlook

Motivation

- πN scattering lengths: key tests of chiral predictions (Tomozawa-Weinberg)

$$a^- = \omega/(8\pi F_\pi^2) \simeq 0.089\ m_\pi^{-1}; \qquad a^+ = 0$$

- the πN scattering lengths: the main input in the GMO determination of πNN coupling constant

$$a^- \quad \text{needed with precision}$$

- a detailed understanding of corrections and high experimental precision necessary

Experimental strong energy shifts in pionic atom hydrogen and deuterium give scattering lengths with spectacular precision (±0.15%), but for corrections (J. Marton Section VII:2)

$$\epsilon^0_{1s} = -\frac{4\pi}{2m}\,\phi_B^2(0)\ a_C \quad ; \text{here } a_C \text{ is the Coulomb scattering length}$$

Our aim is to obtain a clear and quantitative physical picture the e.m. corrections

Background

Two complementary approaches to e.m. corrections

- 1. Effective Field Theory (EFT): systematic, but unknown constants; not physically obvious

- 2. Present approach: less systematic, but also some higher order terms; physically more intuitive; more physically known input included

Our approach gives unambiguously the e.m. corrections generated by g. s. iterations using the pion and nucleon charge distributions instead of point charges in the zero range limit

2 a. Correct initial wave function at r=0

$$\phi_{\mathbf{Bohr}}(\mathbf{0}) \to \phi_{\mathbf{Bohr}}(\mathbf{0})[\mathbf{1}-\alpha \mathbf{m}\langle\mathbf{r}\rangle_{\mathbf{em}} + ..] \to -\mathbf{0.9\%}$$

2 b. Correct energy

$$\omega \to \omega - \mathbf{eV_C(0)}$$

$$\delta\epsilon_{\mathbf{1s;gauge}} = -\frac{\mathbf{4}\pi}{\mathbf{2m}}\phi_{\mathbf{Bohr}}(\mathbf{0})^{\mathbf{2}}(-\alpha\langle\frac{\mathbf{1}}{\mathbf{r}}\rangle_{\mathbf{em}})\mathbf{b}_{\mathbf{h}}^{\pi^- \mathbf{p}} \simeq (-\mathbf{1.0\%})\,\epsilon_{\mathbf{h}}$$

2 c. Correct final wave function

$$\delta\epsilon_{\mathbf{1s;renorm}} = -\mathbf{8}\pi\alpha \mathbf{a}_{\mathbf{h}}^{\mathbf{2}}\,[\mathbf{2}-\gamma+\log \mathbf{2}\alpha - \langle\log \mathbf{mr}\rangle_{\mathbf{em}}]\phi_{\mathbf{Bohr}}(\mathbf{0})^{\mathbf{2}} \simeq (+\mathbf{0.7\%})\,\epsilon_{\mathbf{h}}$$

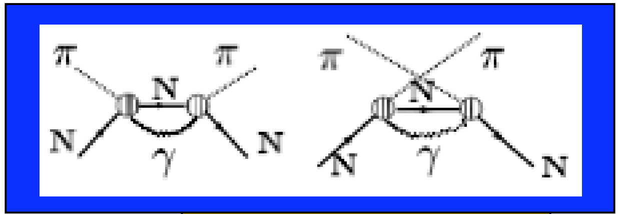
Exact in zero range limit

Excellent approximation even including hadronic range

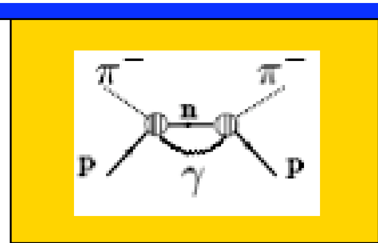
How important are inelastic intermediate states?

Dispersive Corrections

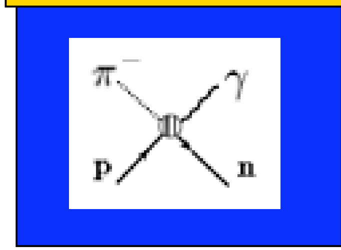
Previous corrections correspond to ground state iterations



Are inelastic states important? $\pi^- N \to \gamma X \to \pi^- N$



Prototype dispersive term $\pi^- p \to \gamma n \to \pi^- p$



Radiative capture width in $\pi^- p$ atom 8% of strong shift!!

The Kroll-Ruderman process:

electric dipole E1 transition radiation induced by p-wave $\pi^- N$ vertex!

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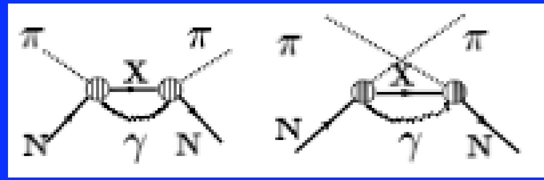
Framework



Key ingredients in the description

Partially Conserved Axial Current PCAC $\partial_\mu \mathbf{J}_{5\mu} = -\mathbf{F}_\pi^2 \mathbf{m}_\pi^2 \phi_\pi$

Gauge invariance $\partial_\mu \to \partial_\mu \pm ie\mathcal{A}_\mu$ (minimal e.m. coupling)

Empirical axial form factor $\mathbf{F_A}(\tilde{\mathbf{q}}^{\,2}) = (\mathbf{1} + \tilde{\mathbf{q}}^{\,2}/\mathbf{M_A^2})^{-2}$ with $\mathbf{M_A} = (\mathbf{960} \pm \mathbf{30})\,\mathbf{MeV}$

Heavy baryon approximation for simplicity and transparency

The threshold condition is important and simplifies the physics

The external threshold pion is replaced by the e. m. axial current vertex $e^\mu(\lambda)J_{5\mu}$

Structure of the dispersive correction at threshold

$$\delta a^{(\gamma)} = ..\frac{\alpha}{F_\pi^2}\int \frac{d^3p}{|\tilde{p}|}\sum_X \left[\frac{...\langle N|J^+_{5\mu}|X\rangle\langle X|J^-_{5\nu}|N\rangle}{E_X + |\tilde{p}| - m_\pi - M_N - i0} + \text{crossed terms}\right]\Bigg|_{\tilde{k}=\tilde{q}=0} e^{*\mu}(\lambda)e^{\nu}(\lambda)$$

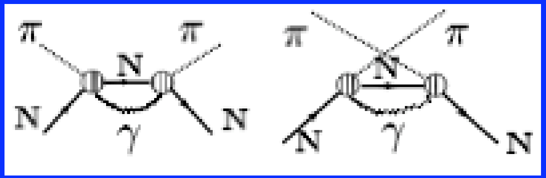
In the soft limit m_π=0 this expression matches exactly the one for the e. m. nucleon mass provided the axial current $J_{5\mu}$ is replaced by the e. m. current J_μ !!

The nucleon e. m. mass is dominated by the nucleon Born term with the charge form factor; other X terms small and given by inelastic electron scattering cross sections

⇒"The Cottingham formula"

The axial formula has N and Δ Born terms; other X terms small (?); test: inelastic (ν,e) scattering

Modification for $m_\pi \neq 0$: change in energy denominators



Dispersive Results

An example: $\pi^- p \Rightarrow \gamma n \Rightarrow \pi^- p$ for $m_\pi = 0$ and $m_\pi \neq 0$

$m_\pi = 0$

$$\delta a_0^{(n\gamma)} = \frac{3\alpha}{8\pi^2}\frac{g_A^2}{F_\pi^2}\int_0^\infty dp F_A^2(p^2) = \frac{15\alpha}{2^8\pi}\frac{g_A^2}{F_\pi^2} M_A \qquad (+3\ \%)$$

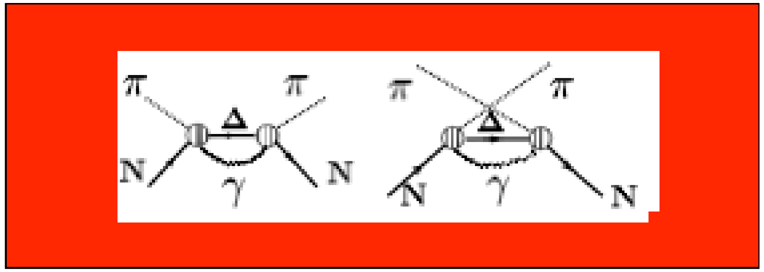
(in % of the $\pi^- p$ scattering length)

$m_\pi \neq 0$

$$\delta a_{m_\pi}^{(n\gamma)} = \frac{3\alpha}{8\pi^2}\frac{g_A^2}{F_\pi^2}\mathcal{P}\int_0^\infty \frac{dp\, p\, F_A^2(p^2)}{p - m_\pi - i0} = \ldots [M_A - \frac{32}{5\pi} m_\pi \left(\ln\frac{m_\pi}{M_A} + \frac{11}{12} + \ldots\right)] \qquad (3\% + 0.4\%)$$

The coefficient of $m_\pi \ln m_\pi$ is identical to 3rd order ChPT (Gasser et al. 2002) !

Dispersive Results



BUT Δ isobar contributions dominate; calculation nearly identical. NΔ mass splitting: 292 MeV a small value!

No NΔ mass splitting and soft pion ⇒ +3% ⇒ +8% isoscalar correction!! HUGE CORRECTION!

Realistic splitting and pion mass: 8% ⇒ 5.1% +0.5%; NΔ splitting main effect LARGE

General result and main uncertainty:

-to the isoscalar 5.1% correction add about ±1% due to $1/M_N$ terms; improvable uncertainty

-the isovector term is small and accurate ⇒ great for testing the Tomozawa-Weinberg relation

Partial comparison to ChPT results

ChPT is a systematic expansion in momentum powers;
e. m. corrections added in to low orders presently

Form factors do not appear in lower orders; heavy baryons assumed

The chiral constants $f_{1,2,3}$ are leading order e. m. terms to be determined empirically

$$M_n^{em} = -e^2F_\pi^2\left(f_1 + f_3\right) \; ; \; M_p^{em} = -e^2F_\pi^2\left(f_1 + f_2 + f_3\right)$$

$$a_{\pi^\pm p}^{em} = -2\pi\alpha\left(f_1 \pm \frac{1}{4} f_2\right)$$

$$(M_p - M_n)^{em} = -2e^2F_\pi^2 f_2 = -\frac{\alpha}{2}\int d^3q \frac{[F_p(q^2)]^2}{q^2}$$

= static heavy nucleon Coulomb energy

f_2 is given by the static $\pi_c p$ finite range Coulomb potential at the origin in our picture

f_1 given by our N,Δ picture :

$$F_\pi^2 f_1^{disp} = -26(1) \text{ MeV}$$

ChPT dimensional estimate $F_\pi^2 |f_1| < 12$ MeV Gasser et al. 2002
Heavy quark model $F_\pi^2 f_1 = -20(2)$ MeV Lyubovitskij et al. 2001

Summary

-The dispersive correction is a dominant contribution to isospin breaking for threshold πN scatteri
It is mainly a branch of well known p-wave πN physics!

-The Δ isobar is essential for the description
(not included in ChPT to 3rd order; Gasser et al.)

-The (N,Δ) Born terms appear to give an accurate description of the dispersion

-The isoscalar violation scales with the axial mass, not with the pion mass
$\Rightarrow$7 times larger than normal

-We obtain both its sign and value reliably

-The isovector breaking is small and accurate

$\Rightarrow$GMO sum rule nearly unchanged

- We have no free parameters

Possible Improvements and Developments

-Elimination of corrections induced kinematically by the heavy baryon limit

-Estimate of contributions from inelastic neutrino cross section contributions (data may exist; talk by A. Bodek, session III.4 Monday)

-The ingredients now exist for a comprehensive quantitative study of threshold πN isospin breaking

-A detailed mapping and interpretation with respect to ChPT results is now possible

-The method can readily be generalized to other pionic atoms

Detailed numerical results for the general πN case

Absolute dispersive corrections in the isospin representation

Contributions to $10^3 m_\pi(\delta a^{n\gamma} + \delta a^{n\Delta})$ in the heavy baryon limit; the uncertainty is the one of M_A

$m_\pi=0;\ \varpi_\Delta=0$	$(3.0(1)_{N\gamma}+5.3(2)_{\Delta\gamma})\ t_3^2$	$=8.3(3)\ t_3^2$
$m_\pi=0;\ \varpi_\Delta\neq 0$	$(3.0(1)_{N\gamma}+2.4(1)_{\Delta\gamma})\ t_3^2$	$=5.4(2)\ t_3^2$
$m_\pi\neq 0;\ \varpi_\Delta=0$	$(2.6(1)_{N\gamma}+4.6(2)_{\Delta\gamma})\ t_3^2 + (-0.8_N+0.7_{\Delta\gamma})\ t_3\tau_3$	$=7.2(2)\ t_3^2 -0.1\ t_3\tau_3$
$m_\pi\neq 0;\ \varpi_\Delta\neq 0$	$(2.6(1)_{N\gamma}+2.5(1)_{\Delta\gamma})\ t_3^2 +(-0.8_{N\gamma}+0.3_{\Delta\gamma})\ t_3\tau_3$	$=5.1(2)\ t_3^2 -0.5\ t_3\tau_3$

(from hep-ph/0503277 and hep-ph/0504258)

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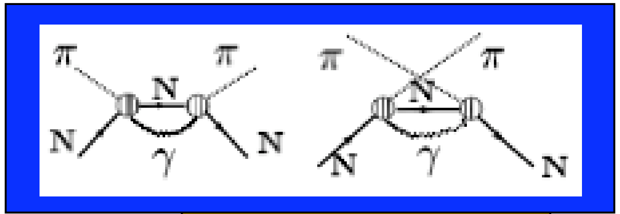

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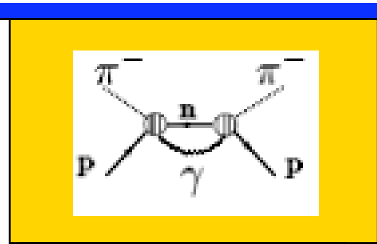

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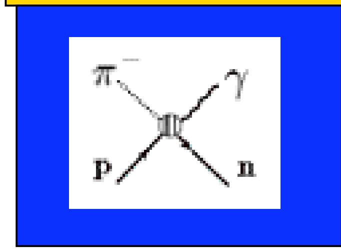

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